THE COIL – WORKPIECE SYSTEM AS A CONTROLLED OBJECT IN CONTINUOUS HEATING

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A method is presented for analytic description of the transfer functions of a controlled object during induction heating in a coil of finite length. If the temperature at the exit from the coil differs slightly from the quasistationary value in the coordinates of the coil, one can use linear approximations (superpositions) and separate the variables for time and coordinates.

Metallurgy and machine construction make considerable use of fast technological processes of hot rolling, heat treatment, and welding via induction heating and with built-in systems for control of the temperature by means of photoelectric temperature transducers. In induction heating, physical features of the method allow the heating rate and metal feed rate to be many times those for heating by gas or electric ovens. The surface temperature of the component at the exit from the coils can be adjusted via the power supply to the coil, the speed of the component, or via the two together. Practical use is found mostly for systems where the coil power is controlled, and instability in the feed rate is considered as additional perturbation affecting the controlled object.

In calculations on such systems one needs a mathematical description of the controlled object, i.e., in terms of transfer functions relating the temperature of the component at the exit from the coil to the basic control and perturbing factors.

Here we consider the transfer functions with respect to the control (coil power) for high-speed induction heating, when a heat transfer along the direction of motion of the metal may be neglected.* The synthesis problem for the automatic control system is then usually one of stabilizing the heating relative to some quasistationary state (in the fixed coordinates of the coil) for small temperature deviations, so one can use linear approximations in deducing the transfer functions, and this allows separation of the time and coordinate variables.

1. Figure 1 shows the half-space of the heated material, which is bounded by the surface $x = 0$, on which the lines show the turns of the coil. First we consider heating by a coil of unlimited length with a known distribution of the heat sources $w(x, y)$. The temperature distribution in the material is described by a two-dimensional Fourier equation:

$$\rho c \frac{\partial \vartheta}{\partial \tau} = \lambda \left(\frac{\partial^2 \vartheta}{\partial x^2} + \frac{\partial^2 \vartheta}{\partial y^2} \right) + w(x, y, \tau), \tag{1}$$

where ϑ is temperature.

We assume that we have a solution for (1) for certain boundary and initial conditions in the complex form

$$\vartheta(x, y, s) = \Phi(x, y, s)\, \bar{p}_{\text{em}}(s), \tag{2}$$

where

$$\bar{p}_{\text{em}} = \int_{-\infty}^{+\infty} dy \int_{0}^{\infty} w(x, y)\, dx \tag{3}$$

is the electromagnetic power per unit length of coil.

*This restriction is not essential for the method described here.

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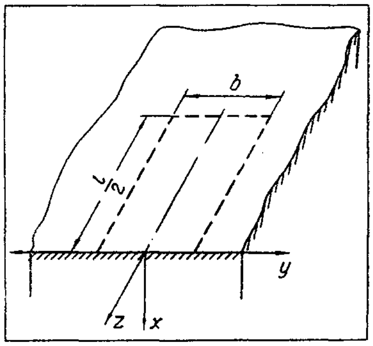

Fig. 1. Zonal heating of a half-space.

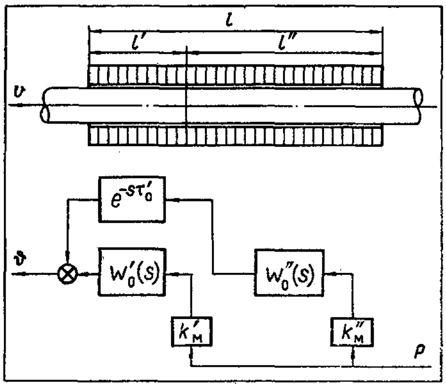

Fig. 2. Structural scheme for controlled object (n = 2).

The solution to (1) may be put as a Duhamel integral (convolution integral) in the region of the real variable for an arbitrarily varying $\bar{p}_{em}(\tau)$:

$$\vartheta(x, y, t) = \frac{d}{dt}\int_0^t h(x, y, \tau)\ \bar{p}_{em}(t-\tau)\,d\tau\,, \tag{4}$$

where

$$h(x, y, \tau) = L^{-1}\left[\ \Phi(x, y, s)\frac{1}{s}\right].$$

We also assume that the part moves along the z axis with a velocity v, while the coil has a restricted length l (Fig. 1).

We neglect changes in the heat-transfer coefficient and the thermophysical parameters of the component under the coil in the direction of motion, as well as edge effects at the boundaries $z = \pm(l/2)$, and then the temperature at the exit from the coil $(z = +(l/2))$ may be put as

$$\vartheta_{out}(x, y, t) = \frac{d}{dt}\int_{t-\tau_0} h(x, y, \tau)\bar{p}_{em}(t-\tau)\,d\tau, \tag{5}$$

where $\tau_0 = l/v$ is the time spent by the workpiece in the coil.

Operational transformation of (5) for $\vartheta_{out}(x, y, 0) = 0$ gives

$$\vartheta_{out}(x, y, s) = s\left[\ \Phi(x, y, s)\frac{1}{s}\right][1-\exp(-s\tau_0)]\bar{p}_{em}(s), \tag{6}$$

whence the transfer function of the controlled object is found in accordance with the coil parameters as

$$W_0(x, y, s) = \frac{\vartheta_{out}(x, y, s)}{P(s)} = k_i s\Phi(x, y, s)\left[\frac{1-\exp(-s\tau_0)}{s\tau_0}\right], \tag{7}$$

where P is the power in the coil, $k_i = \eta_{el}\tau_0/l$, and η_{el} is the electrical efficiency of the coil.

The main interest attaches to the transfer functions in induction heating as calculated for the surface of the workpiece at $x = y = 0$:

$$W_0(s) = \frac{\vartheta_{out}(s)}{P(s)} = k_i s\Phi(s)\left[\frac{1-\exp(-s\tau_0)}{s\tau_0}\right]. \tag{8}$$

If the coil is wide enough and the power is uniformly distributed in the direction of the y axis, the function $\Phi(s)$ may be derived from the solution of the one-dimensional Fourier equation; such solutions have been given [1] for various boundary conditions and with an exponential distribution of the heat sources $w = w_0\exp(-\gamma x)$, where γ is the reciprocal of the half-depth of penetration for the electromagnetic wave with purely surface heating. It was first [1] shown that $\Phi(s)$ may be represented via polynomials that are functions of $\sqrt{s}$; similar representations have been used [2, 3] for the transfer functions on heating in coils of unlimited length for several other characteristic cases. In particular, solutions have been obtained [3] for heating on cylindrical workpieces of radius R. Then the forms used in (7) and (8) for the transfer function enable one to use standard solutions of the one-dimensional or two-dimensional equations for thermal conduction.

We supplement solutions of the type of (2) given in [1-3] by solutions for heating of a sheet of thickness δ comparable with the penetration depth of the electromagnetic wave, when the temperature difference over the cross section of the workpiece is negligible and one needs to take into account only the surface temperature distribution.

Then the energy balance for deviations of the local temperature under the coil is described by

$$\delta\rho c \frac{\partial\vartheta}{\partial\tau} = p_{em}(\tau) - p_l(\tau), \tag{9}$$

where p_l represents the losses, which we represent via Newton's law:

$$p_{p_l} = \alpha\vartheta. \tag{10}$$

We apply operational transformations to (9) with (10) to get

$$\Phi_1(s) = \frac{\vartheta(s)}{p_{em}(s)} = \frac{k_1}{1+sT}, \tag{11}$$

where $k_1 = 1/\alpha$ and $T = \delta\rho c/\alpha$.

For a thermally insulated workpiece with $\alpha = 0$ we have

$$\Phi_2(s) = k_2/s, \tag{12}$$

where $k_2 = 1/\delta\rho c$; substitution of (12) into (8) gives

$$W_0(s) = k_0\left[\frac{1-\exp(-s\tau_0)}{s\tau_0}\right], \tag{13}$$

where $k_0 = k_i k_2/b$.

See [4] for the stability range of an automatic system with the controlled object of (13).

2. The conclusions of section 1 are based on the assumption of a uniform distribution of the power in the coil along the direction of motion (the z axis), which is true only in a restricted number of cases where the heating is produced in a series of heating units or in cyclic heating of workpieces, these being such that the temperature difference between the intake and outlet from a coil is not such as to produce substantial changes in the thermophysical parameters of the workpiece.

If it is necessary to incorporate the power distribution in the coil along the z axis, the coil + workpiece may be represented as a series of zones, within each of which the assumption about a uniform power distribution is reasonably justified.

Figure 2 shows the structural diagram for a controlled object consisting of two zones, where k'_M and k''_M are the coefficients for the power distribution ($k'_M + k''_M = 1$); the transfer function for such an object takes the form

$$W_0(s) = W'_0(s)k'_M + W''_0(s)k''_M\exp(-s\tau'_0), \tag{14}$$

where

$$W'_0(s) = k'_i\, s\Phi'(s)\left[\frac{1-\exp(-s\tau'_0)}{s\tau'_0}\right];$$

$$W''_0(s) = k''_i\, s\Phi''(s)\left[\frac{1-\exp(-s\tau''_0)}{s\tau''_0}\right];$$

$$\tau'_0 = \frac{l'}{v};\quad \tau''_0 = \frac{l''}{v}.$$

If there are n zones, where n = 2, 3, ..., i, then

$$W_0(s) = W'_0(s)k'_M + \sum_2^n W^i_0(s)k^i_M\exp\left(-s\sum_1^{n-1}\tau^i_0\right).$$

This division of the object into zones may be desirable in deriving information from transducers distributed along the heated workpiece.

NOTATION

b is the inductor width, m;
l is the inductor length, m;
v is the velocity, m/sec;
ρ is the density, kg/m^3;
c is the specific heat capacity, J/kg · deg;
λ is the thermal conductivity, W/m · deg;
α is the heat transfer coefficient, W/m^2 · deg;
τ is the current time; sec;
t is the time, sec;
w is the heat source density, W/m^3;
p_{em} is the power W/m^2;
$\bar{p}_{em}$ is the electromagnetic power per unit length of inductor;
P is the inductor power;
γ is the depth of current penetration, m^{-1};
p_l is the loss, W/m^2.

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